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Some properties of primary interval-valued intuitionistic fuzzy M group

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Abstract: **The concept of interval-valued intuitionistic fuzzy M group is extended by introducing primary interval-valued intuitionistic fuzzy M group using this concept primary interval-valued intuitionistic fuzzy M group is defined. We also proved some properties of the above are established.**

Keywords: **Intuitionistic fuzzy set, Interval-valued intuitionistic fuzzy set, Primary interval-valued intuitionistic fuzzy M group.**

1. INTRODUCTION

Ever since an introduction of fuzzy set by L.A.Zadeh [8], the fuzzy concept has invaded almost all branches of mathematics. The concept of IFS and IVIFS was introduced by K.T.Atanassov[1,2] as a generalization of the notion of fuzzy set. K.Chakrabarthy, R.Biswas and S.Nanda [4] discussed union and intersection of IFS. G.Prasannavengeteswari, K.Gunasekaran and .S.Nandakumar [5] introduced the definition of Primary Interval-Valued Intuitionistic Fuzzy M Group (PIVIFMG). In this paper, Some Properties of Primary interval-valued Intuitionistic Fuzzy M Group discussed.

2. PRELIMINARIES

Definition

An IVIFS A over the set E is of the form $A = \{\langle p, M_A(p), N_A(p)\rangle | p \in E\}$, where $M_A(p) \subset [0,1]$ and $N_A(p) \subset [0,1]$ are intervals and $supM_A(p) + supN_A(p) \leq 1$, for every $p \in E$, we note IVIFS A as $A = \{\langle p, [infM_A(p), supM_A(p)], [infN_A(p), supN_A(p)]\rangle | p \in E\}$, we write $[infM_A(p), supM_A(p)] = [\omega_A^-(p), \omega_A^+(p)]$ and $[infN_A(p), supN_A(p)] = [\varpi_A^-(p), \varpi_A^+(p)]$, where $\omega_A^+(p), \varpi_A^+(p), \omega_A^-(p), \varpi_A^-(p)$ are functions from E into [0,1] and $\forall p \in E$, $(\omega_A^-(p) \leq \omega_A^+(p), \varpi_A^-(p) \leq \varpi_A^+(p), \omega_A^+(p) + \varpi_A^+(p) \leq 1)$. Here ω_A^+ and ω_A^- are degree of positive and negative members, ϖ_A^+ and ϖ_A^- degree of positive and negative non-members. we note $\omega_A^-(p) = infM_A(p), \omega_A^+(p) = supM_A(p), \varpi_A^-(p) = infN_A(p), \varpi_A^+(p) = supN_A(p)$.

Definition

Let G be a M group and A be an interval-valued intuitionistic fuzzy subgroup of G, then A is called a PIVIFMG. If for all $p, q \in$ G and $m \in$ M, then either $\omega_A^+(mpq) \leq \omega_A^+(p^x)$ and $\varpi_A^+(mpq) \geq \varpi_A^+(p^x)$, for some $x \in Z_+$ or else $\omega_A^+(mpq) \leq \omega_A^+(q^y)$ and $\varpi_A^+(mpq) \geq \varpi_A^+(q^y)$, for some $y \in Z_+$ and either $\omega_A^-(mpq) \geq \omega_A^-(p^x)$ and $\varpi_A^-(mpq) \leq \varpi_A^-(p^x)$, for some $x \in Z_+$ or else $\omega_A^-(mpq) \geq \omega_A^-(q^y)$ and $\varpi_A^-(mpq) \leq \varpi_A^-(q^y)$, for some $y \in Z_+$.

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Some Properties of primary interval-valued intuitionistic fuzzy M group

Theorem: 1

If R, S and T are primary interval-valued intuitionistic fuzzy M group, then $R \cup (S \cap T) = (R \cup S) \cap (R \cup T)$ is also a primary interval- valued intuitionistic fuzzy M group.

Proof:

Consider $\omega^+_{R\cup(S\cap T)}(mpq) = max\left(\omega^+_R(mpq), \omega^+_{S\cap T}(mpq)\right)$

$$= max\left(\omega^+_R(mpq), min\left(\omega^+_S(mpq), \omega^+_T(mpq)\right)\right)$$

$$= max\left(sup\ M_R\ (mpq), min\left(sup\ M_S\ (mpq), sup\ M_T\ (mpq)\right)\right)$$

$$\leq max\left(sup\ M_R\ (p^x), min\left(sup\ M_S\ (p^x), sup\ M_T\ (p^x)\right)\right)$$

$$= max\left(\omega^+_R(p^x), min\left(\omega^+_S(p^x), \omega^+_T(p^x)\right)\right)$$

$$= min\left(max\left(\omega^+_R(p^x), \omega^+_S(p^x)\right), max\left(\omega^+_R(p^x)\omega^+_T(p^x)\right)\right)$$

$$= min\left(\omega^+_{R\cup S}(p^x), \omega^+_{R\cup T}(p^x)\right)$$

$$= \omega^+_{(R\cup S)\cap(R\cup T)}(p^x)$$

Therefore $\omega^+_{R\cup(S\cap T)}(mpq) \leq \omega^+_{(R\cup S)\cap(R\cup T)}(p^x)$, for some $x \in Z_+$

Consider $\varpi^+_{R\cup(S\cap T)}(mpq) = min\left(\varpi^+_R(mpq), \varpi^+_{S\cap T}(mpq)\right)$

$$= min\left(\varpi^+_R(mpq), max\left(\varpi^+_S(mpq), \varpi^+_T(mpq)\right)\right)$$

$$= min\left(sup\ N_R\ (mpq), max\left(sup\ N_S\ (mpq), sup\ N_T\ (mpq)\right)\right)$$

$$\geq min\left(sup\ N_R\ (p^x), max\left(sup\ N_S\ (p^x), sup\ N_T\ (p^x)\right)\right)$$

$$= min\left(\varpi^+_R(p^x), max\left(\varpi^+_S(p^x), \varpi^+_T(p^x)\right)\right)$$

$$= max\left(min\left(\varpi^+_R(p^x), \varpi^+_S(p^x)\right), min\left(\varpi^+_R(p^x), \varpi^+_T(p^x)\right)\right)$$

$$= max\left(\varpi^+_{R\cup S}(p^x), \varpi^+_{R\cup T}(p^x)\right)$$

$$= \varpi^+_{(R\cup S)\cap(R\cup T)}(p^x)$$

Therefore $\varpi^+_{R\cup(S\cap T)}(mpq) \geq \varpi^+_{(R\cup S)\cap(R\cup T)}(p^x)$, for some $x \in Z_+$

Consider $\omega^-_{R\cup(S\cap T)}(mpq) = min\left(\omega^-_R(mpq), \omega^-_{S\cap T}(mpq)\right)$

$$= min\left(\omega^-_R(mpq), max\left(\omega^-_S(mpq), \omega^-_T(mpq)\right)\right)$$

$$= min\left(inf\ M_R\ (mpq), max\left(inf\ M_S\ (mpq), inf\ M_T\ (mpq)\right)\right)$$

$$\geq min\left(inf\ M_R\ (p^x), max\left(inf\ M_S\ (p^x), inf\ M_T\ (p^x)\right)\right)$$

$$= min\left(\omega^-_R(p^x), max\left(\omega^-_S(p^x), \omega^-_T(p^x)\right)\right)$$

$$= max\left(min\left(\omega^-_R(p^x), \omega^-_S(p^x)\right), min\left(\omega^-_R(p^x), \omega^-_T(p^x)\right)\right)$$

$$= max\left(\omega^-_{R\cup S}(p^x), \omega^-_{R\cup T}(p^x)\right)$$

$$= \omega^-_{(R\cup S)\cap(R\cup T)}(p^x)$$

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Therefore $\omega^{-}_{R\cup(S\cap T)}(mpq) \geq \omega^{-}_{(R\cup S)\cap(R\cup T)}(p^x)$, for some $x \in Z_+$

Consider $\varpi^{-}_{R\cup(S\cap T)}(mpq) = max\left(\varpi^{-}_{R}(mpq), \varpi^{-}_{S\cap T}(mpq)\right)$

$$= max\left(\varpi^{-}_{R}(mpq), min\left(\varpi^{-}_{S}(mpq), \varpi^{-}_{T}(mpq)\right)\right)$$

$$= max\left(inf\ N_R\,(mpq), min\left(inf\ N_S\,(mpq), inf\ N_T\,(mpq)\right)\right)$$

$$\leq max\left(inf\ N_R\,(p^x), min\left(inf\ N_S\,(p^x), inf\ N_T\,(p^x)\right)\right)$$

$$= max\left(\varpi^{-}_{R}(p^x), min\left(\varpi^{-}_{S}(p^x), \varpi^{-}_{T}(p^x)\right)\right)$$

$$= min\left(max\left(\varpi^{-}_{R}(p^x), \varpi^{-}_{S}(p^x)\right), max\left(\varpi^{-}_{R}(p^x), \varpi^{-}_{T}(p^x)\right)\right)$$

$$= min\left(\varpi^{-}_{R\cup S}(p^x), \varpi^{-}_{R\cup T}(p^x)\right)$$

$$= \varpi^{-}_{(R\cup S)\cap(R\cup T)}(p^x)$$

Therefore $\varpi^{-}_{R\cup(S\cap T)}(mpq) \leq \varpi^{-}_{(R\cup S)\cap(R\cup T)}(p^x)$, for some $x \in Z_+$

Therefore $R \cup (S \cap T) = (R \cup S) \cap (R \cup T)$ is also a primary interval- valued intuitionistic fuzzy M group.

Theorem: 2

If R, S and T are primary interval-valued intuitionistic fuzzy M group, then

$R \cap (S \cup T) = (R \cap S) \cup (R \cap T)$ is also a primary interval- valued intuitionistic fuzzy M group.

Proof:

Consider $\omega^{+}_{R\cap(S\cup T)}(mpq) = min\left(\omega^{+}_{R}(mpq), \omega^{+}_{S\cup T}(mpq)\right)$

$$= min\left(\omega^{+}_{R}(mpq), max\left(\omega^{+}_{S}(mpq), \omega^{+}_{T}(mpq)\right)\right)$$

$$= min\left(sup\ M_R\,(mpq), max\left(sup\ M_S\,(mpq), sup\ M_T\,(mpq)\right)\right)$$

$$\leq min\left(sup\ M_R\,(p^x), max\left(sup\ M_S\,(p^x), sup\ M_T\,(p^x)\right)\right)$$

$$= min\left(\omega^{+}_{R}(p^x), max\left(\omega^{+}_{S}(p^x), \omega^{+}_{T}(p^x)\right)\right)$$

$$= max\left(min\left(\omega^{+}_{R}(p^x), \omega^{+}_{S}(p^x)\right), min\left(\omega^{+}_{R}(p^x), \omega^{+}_{T}(p^x)\right)\right)$$

$$= max\left(\omega^{+}_{(R\cap S)}(p^x), \omega^{+}_{(R\cap T)}(p^x)\right)$$

$$= \omega^{+}_{(R\cap S)\cup(R\cap T)}(p^x)$$

Therefore $\omega^{+}_{R\cap(S\cup T)}(mpq) \leq \omega^{+}_{(R\cap S)\cup(R\cap T)}(p^x)$, for some $x \in Z_+$

Consider $\varpi^{+}_{R\cap(S\cup T)}(mpq) = max\left(\varpi^{+}_{R}(mpq), \varpi^{+}_{S\cup T}(mpq)\right)$

$$= max\left(\varpi^{+}_{R}(mpq), min\left(\varpi^{+}_{S}(mpq), \varpi^{+}_{T}(mpq)\right)\right)$$

$$= max\left(sup\ N_R\,(mpq), min\left(sup\ N_S\,(mpq), sup\ N_T\,(mpq)\right)\right)$$

$$\geq max\left(sup\ N_R\,(p^x), min\left(sup\ N_S\,(p^x), sup\ N_T\,(p^x)\right)\right)$$

$$= max\left(\varpi^{+}_{R}(p^x), min\left(\varpi^{+}_{S}(p^x), \varpi^{+}_{T}(p^x)\right)\right)$$

$$= min\left(max\left(\varpi^{+}_{R}(p^x), \varpi^{+}_{S}(p^x)\right), max\left(\varpi^{+}_{R}(p^x), \varpi^{+}_{T}(p^x)\right)\right)$$

$$= min\left(\varpi^{+}_{R\cap S}(p^x), \varpi^{+}_{R\cap T}(p^x)\right)$$

$$= \varpi^{+}_{(R\cap S)\cup(R\cap T)}(p^x)$$

International Journal of Recent Research in Mathematics Computer Science and Information Technology
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Therefore $\varpi^{+}_{R\cap(S\cup T)}(mpq) \geq \varpi^{+}_{(R\cap S)\cup(R\cap T)}(p^x)$, for some $x \in Z_+$

Consider $\omega^{-}_{R\cap(S\cup T)}(mpq) = max\left(\omega^{-}_{R}(mpq), \omega^{-}_{S\cup T}(mpq)\right)$

$$= max\left(\omega^{-}_{R}(mpq), min\left(\omega^{-}_{S}(mpq), \omega^{-}_{T}(mpq)\right)\right)$$

$$= max\left(inf\ M_R\ (mpq), min\left(inf\ M_S\ (mpq), inf\ M_T\ (mpq)\right)\right)$$

$$\geq max\left(inf\ M_R\ (p^x), min\left(inf\ M_S\ (p^x), inf\ M_T\ (p^x)\right)\right)$$

$$= max\left(\omega^{-}_{R}(p^x), min\left(\omega^{-}_{S}(p^x), \omega^{-}_{T}(p^x)\right)\right)$$

$$= min\left(max\left(\omega^{-}_{R}(p^x), \omega^{-}_{S}(p^x)\right), max\left(\omega^{-}_{R}(p^x), \omega^{-}_{T}(p^x)\right)\right)$$

$$= min\left(\omega^{-}_{R\cap S}(p^x), \omega^{-}_{R\cap T}(p^x)\right)$$

$$= \omega^{-}_{(R\cap S)\cup(R\cap T)}(p^x)$$

Therefore $\omega^{-}_{R\cap(S\cup T)}(mpq) \geq \omega^{-}_{(R\cap S)\cup(R\cap T)}(p^x)$, for some $x \in Z_+$

Consider $\varpi^{-}_{R\cap(S\cup T)}(mpq) = min\left(\varpi^{-}_{R}(mpq), \varpi^{-}_{S\cup T}(mpq)\right)$

$$= min\left(\varpi^{-}_{R}(mpq), max\left(\varpi^{-}_{S}(mpq), \varpi^{-}_{T}(mpq)\right)\right)$$

$$= min\left(inf\ N_R\ (mpq), max\left(inf\ N_S\ (mpq), inf\ N_T\ (mpq)\right)\right)$$

$$\leq min\left(inf\ N_R\ (p^x), max\left(inf\ N_S\ (p^x), inf\ N_T\ (p^x)\right)\right)$$

$$= min\left(\varpi^{-}_{R}(p^x), max\left(\varpi^{-}_{S}(p^x), \varpi^{-}_{T}(p^x)\right)\right)$$

$$= max\left(min\left(\varpi^{-}_{R}(p^x), \varpi^{-}_{S}(p^x)\right), min\left(\varpi^{-}_{R}(p^x), \varpi^{-}_{T}(p^x)\right)\right)$$

$$= max\left(\varpi^{-}_{R\cap S}(p^x), \varpi^{-}_{R\cap T}(p^x)\right)$$

$$= \varpi^{-}_{(R\cap S)\cup(R\cap T)}(p^x)$$

Therefore $\varpi^{-}_{R\cap(S\cup T)}(mpq) \leq \varpi^{-}_{(R\cap S)\cup(R\cap T)}(p^x)$, for some $x \in Z_+$

Therefore $R \cap (S \cup T) = (R \cap S) \cup (R \cap T)$ is also a primary interval- valued intuitionistic fuzzy M group.

Theorem: 3

If R and S are primary interval-valued intuitionistic fuzzy M group, then $R \cup (R \cap S) = R$ is also a primary interval- valued intuitionistic fuzzy M group.

Proof:

Consider $\omega^{+}_{R\cup(R\cap S)}(mpq) = max\left(\omega^{+}_{R}(mpq), \omega^{+}_{R\cap S}(mpq)\right)$

$$= max\left(\omega^{+}_{R}(mpq), min\left(\omega^{+}_{R}(mpq), \omega^{+}_{S}(mpq)\right)\right)$$

$$= max\left(sup\ M_R\ (mpq), min\left(sup\ M_R\ (mpq), sup\ M_S\ (mpq)\right)\right)$$

$$\leq max\left(sup\ M_R\ (p^x), min\left(sup\ M_R\ (p^x), sup\ M_S\ (p^x)\right)\right)$$

$$= max\left(\omega^{+}_{R}(p^x), min\left(\omega^{+}_{R}(p^x), \omega^{+}_{S}(p^x)\right)\right)$$

$$= max\left(\omega^{+}_{R}(p^x), \omega^{+}_{S}(p^x)\right)$$

$$= \omega^{+}_{R}(p^x)$$

International Journal of Recent Research in Mathematics Computer Science and Information Technology
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Therefore $\omega^{+}_{R\cup(R\cap S)}(mpq) \leq \omega^{+}_{R}(p^{x})$, for some $x \in Z_{+}$

Consider $\varpi^{+}_{R\cup(R\cap S)}(mpq) = min\big(\varpi^{+}_{R}(mpq), \varpi^{+}_{R\cap S}(mpq)\big)$

$$= min\big(\varpi^{+}_{R}(mpq), max\big(\varpi^{+}_{R}(mpq), \varpi^{+}_{S}(mpq)\big)\big)$$

$$= min\big(sup\, N_{R}\,(mpq), max\big(sup\, N_{R}\,(mpq), sup\, N_{S}\,(mpq)\big)\big)$$

$$\geq min\big(sup\, N_{R}\,(p^{x}), max\big(sup\, N_{R}\,(p^{x}), sup\, N_{S}\,(p^{x})\big)\big)$$

$$= min\big(\varpi^{+}_{R}(p^{x}), max\big(\varpi^{+}_{R}(p^{x}), \varpi^{+}_{S}(p^{x})\big)\big)$$

$$= min\big(\varpi^{+}_{R}(p^{x}), \varpi^{+}_{S}(p^{x})\big)$$

$$= \varpi^{+}_{R}(p^{x})$$

Therefore $\nu^{+}_{R\cup(R\cap S)}(mpq) \geq \varpi^{+}_{R}(p^{x})$, for some $x \in Z_{+}$

Consider $\omega^{-}_{R\cup(R\cap S)}(mpq) = min\big(\omega^{-}_{R}(mpq), \omega^{-}_{R\cap S}(mpq)\big)$

$$= min\big(\omega^{-}_{R}(mpq), max\big(\omega^{-}_{R}(mpq), \omega^{-}_{S}(mpq)\big)\big)$$

$$= min\big(inf\, M_{R}\,(mpq), max\big(inf\, M_{R}\,(mpq), inf\, M_{S}\,(mpq)\big)\big)$$

$$\geq min\big(inf\, M_{R}\,(p^{x}), max\big(inf\, M_{R}\,(p^{x}), inf\, M_{S}\,(p^{x})\big)\big)$$

$$= min\big(\omega^{-}_{R}(p^{x}), max\big(\omega^{-}_{R}(p^{x}), \omega^{-}_{S}(p^{x})\big)\big)$$

$$= min\big(\omega^{-}_{R}(p^{x}), \omega^{-}_{S}(p^{x})\big)$$

$$= \omega^{-}_{R}(p^{x})$$

Therefore $\omega^{-}_{R\cup(R\cap S)}(mpq) \geq \omega^{-}_{R}(p^{x})$, for some $x \in Z_{+}$

Consider $\varpi^{-}_{R\cup(R\cap S)}(mpq) = max\big(\varpi^{-}_{R}(mpq), \varpi^{-}_{R\cap S}(mpq)\big)$

$$= max\big(\varpi^{-}_{R}(mpq), min\big(\varpi^{-}_{R}(mpq), \varpi^{-}_{S}(mpq)\big)\big)$$

$$= max\big(inf\, N_{R}\,(mpq), min\big(inf\, N_{R}\,(mpq), inf\, N_{S}\,(mpq)\big)\big)$$

$$\leq max\big(inf\, N_{R}\,(p^{x}), min\big(inf\, N_{R}\,(p^{x}), inf\, N_{S}\,(p^{x})\big)\big)$$

$$= max\big(\varpi^{-}_{R}(p^{x}), min\big(\varpi^{-}_{R}(p^{x}), \varpi^{-}_{S}(p^{x})\big)\big)$$

$$= max\big(\varpi^{-}_{R}(p^{x}), \varpi^{-}_{S}(p^{x})\big)$$

$$= \varpi^{-}_{R}(p^{x})$$

Therefore $\varpi^{-}_{R\cup(R\cap S)}(mpq) \leq \varpi^{-}_{R}(p^{x})$, for some $x \in Z_{+}$

Therefore $R \cup (R \cap S) = R$ is also a primary interval- valued intuitionistic fuzzy M group.

Theorem: 4

If R and S are primary interval-valued intuitionistic fuzzy M group, then $R \cap (R \cup S) = R$ is also a primary interval- valued intuitionistic fuzzy M group.

Proof:

Consider $\omega^{+}_{R\cap(R\cup S)}(mpq) = min\left(\omega^{+}_{R}(mpq), \omega^{+}_{(R\cup S)}(mpq)\right)$

$$= min\big(\omega^{+}_{R}(mpq), max\big(\omega^{+}_{R}(mpq), \omega^{+}_{S}(mpq)\big)\big)$$

$$= min\big(sup\, M_{R}\,(mpq), max\big(sup\, M_{R}\,(mpq), sup\, M_{S}\,(mpq)\big)\big)$$

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$$\leq min\big(sup\ M_R\ (p^x), max\big(sup\ M_R\ (p^x), sup\ M_S\ (p^x)\big)\big)$$

$$= min\big(\omega_R^+(p^x), max\big(\omega_R^+(p^x), \omega_S^+(p^x)\big)\big)$$

$$= min\big(\omega_R^+(p^x), \omega_S^+(p^x)\big)$$

$$= \omega_R^+(p^x)$$

Therefore $\omega_{R\cap(R\cup S)}^+(mpq) \leq \omega_R^+(p^x)$, for some $x \in Z_+$

Consider $\varpi_{R\cap(R\cup S)}^+(mpq) = max\big(\varpi_R^+(mpq), \varpi_{R\cup S}^+(mpq)\big)$

$$= max\big(\varpi_R^+(mpq), min\big(\varpi_R^+(mpq), \varpi_S^+(mpq)\big)\big)$$

$$= max\big(sup\ N_R\ (mpq), min\big(sup\ N_R\ (mpq), sup\ N_S\ (mpq)\big)\big)$$

$$\geq max\big(sup\ N_R\ (p^x), min\big(sup\ N_R\ (p^x), sup\ N_S\ (p^x)\big)\big)$$

$$= max\big(\varpi_R^+(p^x), min\big(\varpi_R^+(p^x), \varpi_S^+(p^x)\big)\big)$$

$$= max\big(\varpi_A^+(p^x), \varpi_B^+(p^x)\big)$$

$$= \varpi_R^+(p^x)$$

Therefore $\varpi_{R\cap(R\cup S)}^+(mpq) \geq \varpi_R^+(p^x)$, for some $x \in Z_+$

Consider $\omega_{R\cap(R\cup S)}^-(mpq) = max\big(\omega_R^-(mpq), \omega_{R\cup S}^-(mpq)\big)$

$$= max\big(\omega_R^-(mpq), min\big(\omega_R^-(mpq), \omega_S^-(mpq)\big)\big)$$

$$= max\big(inf\ M_R\ (mpq), min\big(inf\ M_R\ (mpq), inf\ M_S\ (mpq)\big)\big)$$

$$\geq max\big(inf\ M_R\ (p^x), min\big(inf\ M_R\ (p^x), inf\ M_S\ (p^x)\big)\big)$$

$$= max\big(\omega_R^-(p^x), min\big(\omega_R^-(p^x), \omega_S^-(p^x)\big)\big)$$

$$= max\big(\omega_R^-(p^x), \omega_S^-(p^x)\big)$$

$$= \omega_R^-(p^x)$$

Therefore $\mu_{R\cap(R\cup S)}^-(mpq) \geq \omega_R^-(p^x)$, for some $x \in Z_+$

Consider $\varpi_{R\cap(R\cup S)}^-(mpq) = min\big(\varpi_R^-(mpq), \varpi_{R\cap S}^-(mpq)\big)$

$$= min\big(\varpi_R^-(mpq), max\big(\varpi_R^-(mpq), \varpi_S^-(mpq)\big)\big)$$

$$= max\big(inf\ N_R\ (mpq), min\big(inf\ N_R\ (mpq), inf\ N_S\ (mpq)\big)\big)$$

$$\leq max\big(inf\ N_R\ (p^x), min\big(inf\ N_R\ (p^x), inf\ N_S\ (p^x)\big)\big)$$

$$= max\big(\varpi_R^-(p^x), min\big(\varpi_R^-(p^x), \varpi_S^-(p^x)\big)\big)$$

$$= max\big(\varpi_R^-(p^x), \varpi_S^-(p^x)\big)$$

$$= \varpi_R^-(p^x)$$

Therefore $\varpi_{R\cap(R\cup S)}^-(mpq) \leq \varpi_R^-(p^x)$, for some $x \in Z_+$

Therefore $R \cap (R \cup S) = R$ is also a primary interval- valued intuitionistic fuzzy M group.

Theorem: 5

If R and S are primary interval-valued intuitionistic fuzzy M group, then $\overline{R} \cap \overline{S} = \overline{R \cup S}$ is also a primary interval- valued intuitionistic fuzzy M group.

International Journal of Recent Research in Mathematics Computer Science and Information Technology
Vol. 11, Issue 2, pp: (23-31), Month: October 2024 – March 2025, Available at: www.paperpublications.org

Proof:

Let R and S be two primary interval-valued intuitionistic fuzzy M group of G.

Consider $p, q \in R \cap S$ and $m \in M$.

Consider $\omega^{+}_{\overline{R \cap S}}(mpq) = min\left(\omega^{+}_{\overline{R}}(mpq), \omega^{+}_{\overline{S}}(mpq)\right)$

$= min(\varpi^{+}_{R}(mpq), \varpi^{+}_{S}(mpq))$

$= min(supN_{R}(mpq), supN_{S}(mpq))$

$\leq min(supN_{R}(p^{x}), supN_{S}(p^{x}))$

$= min\left(\varpi^{+}_{R}(p^{x}), \varpi^{+}_{S}(p^{x})\right)$

$= \varpi^{+}_{R \cup S}(p^{x})$

$= \omega^{+}_{\overline{R \cup S}}(p^{x})$

Therefore $\omega^{+}_{\overline{R \cap S}}(mpq) \leq \omega^{+}_{\overline{R \cup S}}(p^{x})$, for some $x \in Z_{+}$

Consider $\varpi^{+}_{\overline{R \cap S}}(mpq) = max(\varpi^{+}_{\overline{R}}(mpq), \varpi^{+}_{\overline{S}}(mpq))$

$= max(\omega^{+}_{R}(mpq), \omega^{+}_{S}(mpq))$

$= max(supM_{R}(mpq), supM_{S}(mpq))$

$\geq max(supM_{R}(p^{x}), supM_{S}(p^{x}))$

$= max(\omega^{+}_{R}(p^{x}), \omega^{+}_{S}(p^{x}))$

$= \omega^{+}_{R \cup S}(p^{x})$

$= \varpi^{+}_{\overline{R \cup S}}(p^{x})$.

Therefore $\varpi^{+}_{\overline{R \cap S}}(mpq) \geq \varpi^{+}_{\overline{R \cup S}}(p^{x})$, for some $x \in Z_{+}$

Consider $\omega^{-}_{\overline{R \cap S}}(mpq) = max\left(\omega^{-}_{\overline{R}}(mpq), \omega^{-}_{\overline{S}}(mpq)\right)$

$= max(\varpi^{-}_{R}(mpq), \varpi^{-}_{S}(mpq))$

$= max(inf N_{R}(mpq), inf N_{S}(mpq))$

$\geq max(inf N_{R}(p^{x}), inf N_{S}(p^{x}))$

$= max\left(\varpi^{-}_{R}(p^{x}), \varpi^{-}_{S}(p^{x})\right)$

$= \varpi^{-}_{R \cup S}(p^{x})$

$= \omega^{-}_{\overline{R \cup S}}(p^{x})$

Therefore $\omega^{-}_{\overline{R \cap S}}(mpq) \geq \omega^{-}_{\overline{R} \cap S}(p^{x})$, for some $x \in Z_{+}$

Consider $\varpi^{-}_{\overline{R \cap S}}(mpq) = min(\varpi^{-}_{\overline{R}}(mpq), \varpi^{-}_{\overline{S}}(mpq))$

$= min(\omega^{-}_{R}(mpq), \omega^{-}_{S}(mpq))$

$= min(inf M_{R}(mpq), inf M_{S}(mpq))$

$\leq min(inf M_{R}(p^{x}), inf M_{S}(p^{x}))$

$= min(\omega^{-}_{R}(p^{x}), \omega^{-}_{S}(p^{x}))$

$= \omega^{-}_{R \cup S}(p^{x})$

$= \varpi^{-}_{\overline{R \cup S}}(p^{x})$.

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Therefore $\varpi^{-}_{\bar{R}\cap\bar{S}}(mpq) \leq \varpi^{-}_{\overline{R\cup S}}(p^x)$, for some $x \in Z_+$

Therefore, $\bar{R} \cap \bar{S} = \overline{R \cup S}$ is a primary interval-valued intuitionistic fuzzy M group of G. □

Theorem: 6

If R and S are primary interval-valued intuitionistic fuzzy M group, then $\bar{R} \cup \bar{S} = \overline{R \cap S}$ is also a primary interval- valued intuitionistic fuzzy M group.

Proof:

Let R and S be two primary interval-valued intuitionistic fuzzy M group of G.

Consider $\omega^{+}_{\bar{R}\cup\bar{S}}(mpq) = max\left(\omega^{+}_{\bar{R}}(mpq), \omega^{+}_{\bar{S}}(mpq)\right)$

$= max(\varpi^{+}_{R}(mpq), \varpi^{+}_{S}(mpq))$

$= max(supN_R(mpq), supN_S(mpq))$

$\leq max(supN_R(p^x), supN_S(p^x))$

$= max\left(\varpi^{+}_{R}(p^x), \varpi^{+}_{S}(p^x)\right)$

$= \varpi^{+}_{R\cap S}(p^x)$

$= \omega^{+}_{\overline{R\cap S}}(p^x)$

Therefore $\omega^{+}_{\bar{R}\cup\bar{S}}(mpq) \leq \omega^{+}_{\overline{R\cap S}}(p^x)$, for some $x \in Z_+$

Consider $\varpi^{+}_{\bar{R}\cup\bar{S}}(mpq) = min(\varpi^{+}_{\bar{R}}(mpq), \varpi^{+}_{\bar{S}}(mpq))$

$= min(\omega^{+}_{R}(mpq), \omega^{+}_{S}(mpq))$

$= min(supM_R(mpq), supM_S(mpq))$

$\geq min(supM_R(p^x), supM_S(p^x))$

$= min(\omega^{+}_{R}(p^x), \omega^{+}_{S}(p^x))$

$= \omega^{+}_{R\cap S}(p^x)$

$= \varpi^{+}_{\overline{R\cap S}}(p^x)$.

Therefore $\varpi^{+}_{\bar{R}\cup\bar{S}}(mpq) \geq \varpi^{+}_{\overline{R\cap S}}(p^x)$, for some $x \in Z_+$

Consider $\omega^{-}_{\bar{R}\cup\bar{S}}(mpq) = min\left(\omega^{-}_{\bar{R}}(mpq), \omega^{-}_{\bar{S}}(mpq)\right)$

$= min(\varpi^{-}_{R}(mpq), \varpi^{-}_{S}(mpq))$

$= min(inf N_R(mpq), inf N_S(mpq))$

$\geq min(inf N_R(p^x), inf N_S(p^x))$

$= min\left(\varpi^{-}_{R}(p^x), \varpi^{-}_{S}(p^x)\right)$

$= \varpi^{-}_{R\cap S}(p^x)$

$= \omega^{-}_{\overline{R\cap S}}(p^x)$

Therefore $\omega^{-}_{\bar{R}\cup\bar{S}}(mpq) \geq \omega^{-}_{\overline{R\cap S}}(p^x)$, for some $x \in Z_+$

Consider $\varpi^{-}_{\bar{R}\cup\bar{S}}(mpq) = max(\varpi^{-}_{\bar{R}}(mpq), \varpi^{-}_{\bar{S}}(mpq))$

$= max(\omega^{-}_{R}(mpq), \omega^{-}_{S}(mpq))$

$= max(inf M_R(mpq), inf M_S(mpq))$

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$$\leq max(inf M_R(p^x), inf M_S(p^x))$$

$$= max(\omega_R^-(p^x), \omega_S^-(p^x))$$

$$= \omega_{R\cap S}^-(p^x)$$

$$= \varpi_{\overline{R\cap S}}^-(p^x).$$

Therefore $\varpi_{\bar{R}\cup\bar{S}}^-(mpq) \leq \varpi_{\overline{R\cap S}}^-(p^x)$, for some $x \in Z_+$

Therefore $\bar{R} \cup \bar{S} = \overline{R \cap S}$ is a primary interval-valued intuitionistic fuzzy *M* group of *G*.

3. CONCLUSION

In this paper the main idea of primary interval-valued intuitionistic fuzzy M group is a new algebraic structures of fuzzy algebra and it is used through the some properties of the above. We believe that our ideas can also applied for other algebraic system.

REFERENCES

[1] Atanassov, K. T. (1999). *Intuitionistic Fuzzy Sets:Theory and Application*, Springer Physica-Verlag.

[2] Atanassov, K. T. (2020). *Interval-Valued Intuitionistic Fuzzy Sets*, Springer Cham.

[3] Balasubramanian, A., Muruganantha Prasad, K. L., &Arjunan, K. (2015). Bipolar Interval Valued Fuzzy Subgroups of a Group, *Bulletin of Mathematics and Statistics Research*,3(3), 234–239.

[4] Chakrabarthy, K., Biswas, R., & Nanda, S. (1997). A note on union and intersection of intuitionistic fuzzy sets. *Notes on Intuitionistic Fuzzy Sets*, 3(4), 34–39.

[5] Prasannavengeteswari, G., Gunasekaran, K., &Nandakumar, S. (2022). primary interval-valued intuitionistic fuzzy M group. *Notes on Intuitionistic Fuzzy Sets*, 28(2), 120-131.

[6] Rosenfeld, A. (1971). Fuzzy Groups, *Journal of Mathematical Analysis and Its Application*, 35, 512–517.

[7] Zadeh, L. A. (1965). Fuzzy sets.*Information and Control*, 8, 338–353.

[8] Zhang, W. R. (1998). Bipolar fuzzy sets. *Proceeding of FUZZ-IEEE*, 835–840.

[9] Zimmermann, H. J. (1985). *Fuzzy Set Theory and Its Applications*, Kluwer-Nijhoff Publishing Co.